Modeling Plasma Wake Field Accelerator

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**Why Plasma?**

1-D plasma density wave

\[ V_{ph} = c \]

Gauss’ Law

\[ \nabla \cdot E \sim \imath k_p E = -4\pi e n_1 \]

\[ k_p = \omega_p / V_{ph} \approx \omega_p / c \]

\[ n_1 \sim n_o \]

\[ \Rightarrow eE \sim 4\pi e n_o e^2 c / \omega_p = mc \omega_p \]

or \[ eE \sim \sqrt{\frac{n_o}{10^{16} \text{cm}^{-3}}} \] 10 GeV/m

\(~1000~\text{times larger than the conventional accelerators}\)
How to Make a Plasma Wake Field?

Nonlinear Process

LWFA:
- Drive Beam
- Trailing Beam
- Wake: phase velocity = driver velocity ($V_g$ or $V_{beam}$)

PWFA:

LWFA: Tajima and Dawson 1979
PWFA: Chen, Dawson et al., 1985

Plasma simulation has greatly impacted on PBA research.
Simulation of PWFA

Beam Particles: $10^8$−$10^9$

Plasma Length: $\sim 1$ m

Moving Window

Plasma Particles: $>10^{10}$

Maxwell's Eqns

\[
\begin{align*}
\nabla \times \vec{E} & = -\frac{\partial \vec{B}}{\partial t} \\
\nabla \times \vec{B} & = \frac{\partial \vec{E}}{\partial t} + \vec{J} \\
\n\nabla \cdot \vec{E} & = \rho \\
\n\nabla \cdot \vec{B} & = 0
\end{align*}
\]

All particles move self-consistently

~ 500 µm
Box and Cell Size

3D or 2D r-z with moving window

Box Size: Large Enough to minimize the boundary effects.

Cell Size: Resolve the plasma wave length.

\[ \leq 0.05k_p^{-1} \]
Define the density profile using a math function.
Field Ionized Plasma
The focal length of the beam

\[ \beta^* = \gamma \frac{\sigma_r^2}{\epsilon_N} \]

\[ \sigma_r = \sigma_{r0} \sqrt{1 + \frac{(s - s_0)^2}{\beta^*^2}} \]

Twiss Parameter:
\[ \gamma x'^2 + 2\alpha xx' + \beta x^2 = \epsilon \]

In the Vacuum:
\[ \gamma = \frac{1}{\beta^*}, \quad \beta = \beta^*(1 + \alpha^2), \quad \alpha = -\frac{s - s_0}{\beta^*} \]
Beam Density: \[ n_b = n_{b0} \exp\left(-\frac{r^2}{2\sigma_r^2}\right) \exp\left(-\frac{z^2}{2\sigma_z^2}\right) \]

Transverse Phase Space:
\[ \sim \exp\left(-\frac{x^2}{2\sigma_{x0}^2}\right) \exp\left(-\frac{v^2}{2\sigma_{v0}^2}\right) \]

Transverse Phase Space at \( s^* = z - z_0 \):
\[ \sim \exp\left[-\frac{x^2}{2\sigma_{x0}^2(1 + s^*^2/\beta^2)}\right] \exp\left[-\frac{(v - \frac{s^*cx}{\beta^2 + s^*^2})^2}{2\sigma_{v0}^2/(1 + s^*^2/\beta^2)}\right] \]

\[ \bar{\sigma}_x = \sigma_{x0}\sqrt{1 + s^*^2/\beta^2} \quad \bar{\sigma}_v = \sigma_{v0}/\sqrt{1 + s^*^2/\beta^2} \]

Good for Osiris Initialization!
QuickPIC Open Source

Boundary Condition
Conducting
Interpolation Order
1st

MPI or Shared Memory
MPI + OpenMP

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Fortran 2003
Object Oriented

Fortran 77
Fortran 90

Github QuickPIC-OpenSource