Boris correction: exact and approximate

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Outline

• OSIRIS current deposits
  1. Charge-conserving current deposit
     • Standard in OSIRIS, satisfies continuity equation
  2. Direct current deposit
     • Use particle positions/shape functions
     • Less noisy, does not satisfy continuity equation

• Divergence cleaning (Boris correction)
  1. Iterative multigrid solver
  2. Exact spectral solver
  3. Exact spectral and finite-difference solver
Particle push

\[
\frac{dx}{dt}_k = v_k \\
\frac{dp}{dt}_k = q(E_k + v_k \times B_k)
\]

Charge/current deposition

Calculate \( \rho_{ij} \) and \( j_{ij} \) from the \( x_k \) and \( p_k \).

Field interpolation

Interpolate \( E_{ij} \) and \( B_{ij} \) onto the \( x_k \).

Field solve

\[
\frac{\partial B_{ij}}{\partial t} = -\nabla \times E_{ij} \\
\frac{\partial E_{ij}}{\partial t} = c^2 \nabla \times B_{ij} - \mu_0 c^2 j_{ij}
\]

\( k \): continuous (particles)  
\( ij \): gridded (fields)
• Gauss’s law not explicitly solved in OSIRIS

• If initial $\rho$ and $E$ satisfy Gauss’s law, then continuity equation guarantees Gauss’s law at future times

\[
\frac{\partial \rho_{ij}}{\partial t} + \nabla \cdot \mathbf{j}_{ij} \neq 0
\]

\[\Downarrow\]

\[
\nabla \cdot \mathbf{E}_{ij} \neq \frac{\rho_{ij}}{\varepsilon_0}
\]
Current deposits

• Charge-conserving current deposit inherently satisfies continuity equation (and thus Gauss’s law)

• Direct deposit—uses particle shapes

Diagram:
- Default
- Average particle shapes
Boris correction

• Solve Poisson’s equation to correct electric field

\[ \nabla^2 \psi = \nabla \cdot E - \frac{\rho}{\epsilon_0} \equiv \rho' \]

• where \( \rho' \) is the residual charge error. Then

\[ E' = E - \nabla \psi \]

\[ \nabla \cdot E' = \frac{\rho}{\epsilon_0} \]
Iterative multigrid solver: V-cycle

- Solve the matrix equation
  \[ Ax = b \quad (\text{i.e., } \nabla^2 \psi = \rho') , \]

- Start with a guess of \( \psi = 0 \) (\( x = 0 \))
- Compute residual (\( r = Ax - b \))
- Restrict onto coarser grid (\( r \rightarrow r_1 \))
- Smooth solution using point-Jacobi (\( A_1e_1 = r_1 \))
- Direct matrix solve of small problem (\( A_Ne_N = r_N \))
- Interpolate onto finer grid (\( e_N \rightarrow e_{N-1}^* \))
- Correct the solution (\( e_{N-1} = e_{N-1} - e_{N-1}^* \), \( x = x - e_0^* \))
- Iterate
Iterative multigrid solver
Exact spectral solver

• Assuming Fourier behavior, ODE $\rightarrow$ algebraic equation

$$\nabla^2 \psi = \rho' \rightarrow -k^2 \psi = \rho'$$

• However, finite-difference derivative is approximate:

$$\mathcal{F} \left\{ \frac{\psi(x + h) - 2\psi(x) + \psi(x - h)}{h^2} \right\} = -\frac{4}{h^2} \sin^2 \left( \frac{\pi k}{N} \right)$$

• One dimension ($x$) must be entirely local
Hybrid spectral/finite-difference

• 3D solver
  • Spectral in \( x \) and \( y \)
  • Finite-difference in \( z \)

\[
\frac{\psi(z + h) - (2 + k_x^2 + k_y^2)\psi(z) + \psi(z - h)}{h^2} = \rho(z)
\]

• Should be more consistent than fully spectral
Speedup

• Compared to point-Jacobi, multigrid is much faster
• Spectral methods should be even faster
Test: stimulated Raman scattering

Without parallel multigrid

Charge-conserving deposit

Direct deposit

With parallel multigrid

Kyle Miller, OSIRIS Workshop 2017, 12
Test: stimulated Raman scattering

Charge-conserving deposit

Without parallel multigrid

With parallel multigrid

Direct deposit
Use direct current deposit

• `#define DIRECT_DEPOSIT`
  • in os-config.h

• `#define AVG_SHAPES`
  • in os-config.h
Use Boris correction

• if_boris = .true.,

• boris_solver = ’multigrid’, ‘fft’, ‘hybrid’, or ’point-Jacobi’,
  • in el_mag_fld of input file

• Other options:
  • boris_tol = 1.e-14,
  • boris_n = 4,
Important files

- Multigrid:
  - pmg/* (esp m_parallel_multigrid_solver.f90)
  - os-emf-es-solver-pmg.f03
  - os-emf-marder.f90
  - os-emf-math.f03
  - os-vdf-math.f90

- Exact Boris
  - boris/*

- Branch parallel-multigrid on Github
Conclusions

• Implementations:
  • Direct current deposit
    • Using average position or average particle shapes
  • Boris correction for use with direct deposit
    • Using iterative multigrid, exact spectral, or point-Jacobi

• Uses
  • Less noisy current deposit
  • Solver may be used for other initialization purposes

• Future work
  • Finish implementation, run more physics test cases